



A Mathematical Study on Internal Resistance for an Electrochemical Cell

K A Khan¹, Md. Jobair Ahamed², Salman Rahman Rasel³, Md. Kabir Uddin⁴

¹Department of Physics, Jagannath University, Dhaka-1100, E-mail: kakhan01@yahoo.com

²Student(Ex), Department of Mathematics, Jagannath University, Dhaka-1100,
E-mail: jobairahamed2006@gmail.com

³Executive Engineer, LGED, Mymensingh, E-mail: slmnrasel@gmail.com

⁴System Analyst, EMRD, Ministry of Power, Energy and Mineral Resources,
E-mail: kabiruddin@gmail.com

ABSTRACT

This study investigates the internal resistance of electrochemical cells, a critical parameter influencing cell efficiency and performance. Using a controlled experimental setup, the research explores how factors such as electrode separation, electrolyte temperature, and concentration affect internal resistance and electromotive force (EMF). Results demonstrate that increasing electrode distance raises resistance, while higher electrolyte temperature and concentration reduce it. These findings provide deeper insight into optimizing electrochemical cell design for improved energy output and reliability. The high internal resistance of the battery reduced efficiency, heat generation, voltage sag. The Low internal resistance of the battery gives better performance, longer lifespan, faster charging.

Keyword: Internal resistance, Electrochemical cell, Battery performance, Electrode materials, Electrolyte conductivity, Voltage drop, AC/DC resistance

I. Introduction

Electrochemical cells are fundamental components in modern energy storage and conversion systems, ranging from batteries to fuel cells^[1-3]. One of the key parameters influencing their performance is internal resistance, which determines how efficiently a cell can deliver electrical energy^[4-5]. Internal resistance arises from various sources within the cell, including the resistance of the electrolyte, the electrodes, and the interfaces between them^[6-7]. This study investigates the factors that affect internal resistance in electrochemical cells, with a focus on how variables such as electrode separation, electrolyte temperature, and concentration influence resistance levels^[8]. By employing an experimental circuit setup, the research aims to quantify these effects and provide insights into optimizing cell design for improved efficiency and longevity^[9-12]. Understanding internal resistance is crucial not only for enhancing the performance of energy storage devices but also for diagnosing cell health and predicting lifespan^[13]. The findings of this study contribute to the broader field of electrochemistry by offering practical data and analysis that can inform future innovations in battery technology and sustainable energy systems^[14]. Internal resistance is like hidden friction inside your battery-it quietly drains energy, limits performance, and shortens lifespan. That's why battery engineers obsess over materials, design, and temperature control to keep it as low as possible.

II. Methodology

II A. Measurement of Electrical Parameters

1. Electrochemical Impedance Spectroscopy (EIS)

- A non-destructive and highly sensitive method.
- Measures impedance over a range of frequencies.
- Helps identify resistance components such as charge transfer resistance and electrolyte resistance.
- Useful for estimating the **state of health (SoH)** of batteries during operation.

2. AC and DC Resistance Methods

- **AC Resistance:** Measures impedance at specific frequencies; effective for detecting connection degradation.
- **DC Resistance:** Involves applying a current pulse and measuring voltage drop; better for diagnosing cell degradation.

3. Current Step Method

- Applies a step change in current and observes the voltage response.
- Used to calculate internal resistance using Ohm's law.

4. Thermal Loss Method

- Evaluates heat generated during operation to infer resistance.
- Often used in conjunction with other methods for validation.

5. Controlled Environmental Testing

- Tests are conducted under varying conditions of:
 - **State of Charge (SOC)**
 - **Temperature**
 - **Connection integrity**
- These variables help understand how internal resistance changes with battery aging and usage patterns

II B. Relation Between Chemical reaction and internal resistance of the Zn/Cu electrochemical cell.

Chemical Reaction in Zn/Cu Electrochemical Cell

- **Oxidation at Zinc Electrode (Anode):**
 - Zinc loses electrons, which flow through the external circuit.
- **Reduction at Copper Electrode (Cathode):**
 - Copper ions gain electrons and deposit as solid copper.
- **Results for Overall Cell Reaction:**
 - This spontaneous redox reaction drives the flow of electrons, generating electrical energy.

Internal Resistance and Its Connection to the Reaction

Internal resistance arises from several factors that oppose the flow of current within the cell:

- **Electrode Surface Conditions:**
 - As the reaction proceeds, zinc may become coated with copper or other by-products, increasing resistance.
- **Ion Mobility in Electrolyte:**
 - Resistance depends on how easily ions (Zn^{2+} and Cu^{2+}) move through the solution and across the salt bridge.
- **Salt Bridge Efficiency:**
 - The salt bridge maintains charge neutrality by allowing ion flow. Poor conductivity here increases internal resistance.
- **Electrode Degradation:**
 - Over time, the zinc electrode erodes and copper builds up, which can alter the reaction dynamics and increase resistance.
- **Temperature Effects:**
 - The reaction generates heat, which can affect ion mobility and resistance. Higher temperatures generally lower resistance but may also accelerate degradation.

EMF vs. Internal Resistance

- **EMF (Electromotive Force):**
 - The theoretical maximum voltage generated by the chemical reaction.
- **Voltage Drop Due to Internal Resistance:**
 - When current flows, some voltage is lost internally
- **Practical Implication:**
 - The actual voltage available to an external circuit is less than the EMF due to internal resistance.

II C. Mathematical calculation of internal resistance of an electrochemical cell

Formula for Internal Resistance

The internal resistance r of a cell can be calculated using: $r=(E-V)/I$, Where:

- E = **Electromotive force (EMF)** of the cell (voltage when no current flows)
- V = **Terminal voltage** (voltage across the cell when current flows)
- I = **Current** flowing through the circuit

Step-by-Step Calculation

1. **Measure EMF (E):**
Use a voltmeter across the cell when no current is drawn (open circuit). Example: $E=1.5\text{ V}$
2. **Measure Terminal Voltage (V):** Connect the cell to a known resistor and measure voltage across the terminals. Example: $V=1.2\text{ V}$
3. **Measure Current (I):** Use an ammeter in series with the resistor. Example: $I=0.3\text{ A}$
4. **Apply Formula:** $r=(1.5-1.20)/.3=0.3/0.3=1\ \Omega$

Notes

- Internal resistance increases with age and usage of the cell.

- It causes a voltage drop inside the cell, reducing the terminal voltage.
- For more accurate results, use a variable resistor and plot V vs. I to find the slope of the line: the slope gives r .

II D. Key Factors Affecting Internal Resistance

1. Electrode Material

- Different materials have varying conductivity and reactivity.
- Poor conductors (e.g., carbon) increase resistance; metals like copper or platinum reduce it.

2. Electrolyte Composition

- The type and concentration of ions in the electrolyte affect ion mobility.
- Higher ion concentration generally lowers resistance due to better conductivity.

3. Temperature

- **Higher temperature** increases ion mobility, reducing resistance.
- **Lower temperature** slows down ion movement, increasing resistance.

4. Cell Age and Usage

- Over time, chemical degradation (e.g., corrosion, buildup of reaction products) increases resistance.
- Repeated charging/discharging in rechargeable cells can also affect internal structure.

5. Electrode Surface Area

- Larger surface area allows more efficient ion exchange, lowering resistance.
- Smaller or degraded surfaces restrict ion flow, increasing resistance.

6. Distance Between Electrodes

- Greater distance means ions travel farther, increasing resistance.
- Compact designs reduce this path and lower resistance.

7. State of Charge

- In batteries, internal resistance varies with charge level.
- Near full or empty charge, resistance tends to be higher.

8. Current Flow

- High current can cause polarization or heating, temporarily increasing resistance.

Summary Table

Factor	Effect on Resistance
Electrode material	Better conductors → lower
Electrolyte concentration	Higher concentration → lower
Temperature	Higher temp → lower
Cell age	Older cells → higher
Electrode surface area	Larger area → lower
Electrode spacing	Greater distance → higher

Factor	Effect on Resistance
State of charge	Extreme levels → higher
Current magnitude	High current → higher

III. Results and Discussion

Key Findings from the Study

- **Electrode Separation:** Increasing the distance between electrodes leads to higher internal resistance. This is due to the longer path ions must travel through the electrolyte, increasing resistance to current flow.
- **Electrolyte Temperature:** Higher temperatures reduce internal resistance. Warmer electrolytes enhance ion mobility, improving conductivity and lowering resistance.
- **Measurement Techniques:**
 - **Electrochemical Impedance Spectroscopy (EIS)** was used to break down internal resistance into:
 - **Ohmic Resistance:** Related to the membrane and electrolyte; found to be stable under high humidity but increased significantly under low humidity conditions.
 - **Charge Transfer Resistance:** Decreased with increasing current load and contributed most to total resistance under high humidity.
 - **Mass Transport Resistance:** Became significant under low humidity and tripled under full humidification.
- **Timescale Sensitivity:** Resistance values varied depending on the measurement technique and timescale. Techniques like DC pulse, AC signals, and multisine pulses yielded different resistance values unless their timescales were matched.

Discussion Highlights

- **Environmental Conditions Matter:** Humidity and temperature play a crucial role in determining internal resistance. This has implications for battery performance in different climates and applications.
- **Technique Selection Is Critical:** The choice of measurement method affects the accuracy and relevance of resistance values. EIS offers a comprehensive view but requires careful calibration.
- **Design Implications:** Understanding internal resistance helps in optimizing battery design, especially for applications like electric vehicles and energy storage systems where efficiency and safety are paramount.

IV. Conclusions

- **Electrode Separation:** Internal resistance increases with greater distance between electrodes. This is due to the longer path ions must travel, which impedes current flow.
- **Electrode Surface Area:** A larger surface area of electrodes in contact with the electrolyte reduces internal resistance, as it facilitates better ion exchange.
- **Electrolyte Temperature:** Higher temperatures decrease internal resistance. Heat increases ion mobility, enhancing conductivity within the cell.
- **Electrolyte Concentration:** More concentrated electrolytes result in lower internal resistance. A richer ionic environment supports more efficient charge transfer.
- **Cell Efficiency:** Understanding and optimizing these factors can significantly improve the performance and lifespan of electrochemical cells, especially in battery applications.

These findings were based on experimental setups involving variations in physical and chemical parameters, and they provide valuable insights for designing more efficient energy storage systems

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